# Illinois Model Curriculum Scope & Sequence

**Grade/Course:** 6th Grade Math

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| 1) Fractions and Decimals   | 6.NS.1  
6.NS.2  
6.NS.3 | 6.G.1  
6.G.4 | 3-4 weeks             |
| 2) Ratios, Rates, and Proportions | 6.RP.1  
6.RP.2  
6.RP.3  
6.EE.9 | | 6-7 weeks             |
| 3) Rational Numbers         | 6.NS.5  
6.NS.6  
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6.NS.8  
6.G.3 | | 4-5 weeks             |
| 4) Expressions              | 6.EE.1  
6.EE.2  
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6.NS.4 | | 3-4 weeks             |
| 5) Equations and Inequalities | 6.EE.5  
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6.EE.7  
6.EE.8 | | 4-5 weeks             |
| 6) Geometry                 | 6.G.1  
6.G.2  
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| 7) Statistics               | 6.SP.1  
6.SP.2  
6.SP.3  
6.SP.4  
6.SP.5 | 6.RP.3b | 2-3 weeks             |
| 8) Formulas and Graphs      | 6.EE.9  
6.NS.8  
6.SP.4  
6.G.2 | 6.G.3 | 1-2 weeks             |
Connections to Previous Learning:
By the end of grade 4, students are expected to fluently add, subtract, multiply and divide multi-digit whole numbers. In grade 5, students are expected to fluently add, subtract, multiply and divide decimals to hundredths. 6th graders will extend these skills to include any decimals using standard algorithms for each operation.

Focus within the Grade Level:
Sixth graders will extend these skills to dividing a fraction by a fraction. There is not one single standard algorithm; however, a standard algorithm is a strategy that works every time. Some well-known algorithms include partial sums, differences, products and quotients, tradition algorithms for each operation (including regrouping). However, some students develop algorithms that are efficient, allow for fluent calculation and demonstrate understanding of both the value and the operations. If students can justify their reasoning for an algorithm, it may be categorized as a “standard algorithm.”

Connections to Subsequent Learning:
Students can add, subtract, multiply and divide using decimals and fractions to solve complex application problems. In seventh grade students will use percents and scale factors to determine percent of increase or percent of decrease, discounts and markups. They will also use the understanding of area and surface area to solve problems involving area, volume and surface area of 2-D and 3-D objects in 7th grade.

Desired Outcomes

Standard(s):

Apply and extend previous understandings of multiplication and division to divide fractions by fractions
- 6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?

Compare fluently with multi-digit numbers and find common factors and multiples
- 6.NS.2 Fluently divide multi-digit numbers using the standard algorithm.
- 6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
Supporting Standards:
Solve real-world and mathematical problems involving area, surface area, and volume
- 6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
- 6.G.4 Represent three-dimensional figures using nets made up of rectangles and triangles. Use the nets to find surface areas of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

WIDA Standard:  (English Language Learners)
English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.
English language learners benefit from:
- the opportunity to use visual and concrete models in order to understand and apply fraction and decimal concepts and language.
- explicit vocabulary instruction regarding fractions and decimals.

Understandings:  Students will understand that ...
- The two types of division – quotative (partitive) and measurement are applied to fractions and decimals as well as to whole numbers.
- Multiplication and division are inverse operations.
- The relationship of the location of the digits and the value of the digits is part of understanding multi-digit operations.
- Division can be represented using multiple formats (manipulatives, diagrams, real-life situations, equations).
- Operations on decimals and whole numbers are based upon place value relationships.
- Problems of area of polygons can be solved by composing and decomposing the polygons.

Essential Questions:
- How is division related to realistic situations and to other operations?
- What role does place value play in multi-digit operations?
- How can division be represented and interpreted?
- In what ways can the area of a net be determined?
Mathematical Practices: (Practices to be explicitly emphasized are indicated with an *.)

1. **Make sense of problems and persevere in solving them.** Students make sense of real-world fraction and decimal problem situations by representing the context in tactile and/or virtual manipulatives, visual, or algebraic models.

2. **Reason abstractly and quantitatively.** Students will apply the constructs of multiplication and division of rational numbers to solve application problems, including finding the area of nets.

3. **Construct viable arguments and critique the reasoning of others.** Students construct and critique arguments regarding the portion of a whole as represented in the context of real-world situations.

4. **Model with mathematics.** Students will model real-world situations to show multiplication and division of fractions and decimals.

5. **Use appropriate tools strategically.** Students will use visual or concrete tools for division of fractions with understanding. (Such as fraction square or circle pieces, fraction equivalence towers, bar models, and number line diagrams.

6. **Attend to precision.** Students attend to the language of problems to determine appropriate representations and operations for solving real-world problems. In addition, students attend to the units of measure used in real-world problems.

7. **Look for and make use of structure.** Students examine the relationship of rational numbers to the number line and the place value structure as related to multi-digit operations. They also use their knowledge of problem solving structures to make sense of word problems.

8. **Look for and express regularity in repeated reasoning.** Students demonstrate repeated reasoning when dividing fractions by fractions by fractions and see the inverse relationship to multiplication.

**Prerequisite Skills/Concepts:**

**Students should already be able to:**

- Add, subtract and multiply fractions.
- Divide fractions by whole numbers and whole numbers by fractions.
- Use area models for fraction or decimal computation situations.
- Fluently add, subtract, multiply and divide whole numbers.
- Use concepts of area, perimeter and volume to solve problems with whole numbers.

**Advanced Skills/Concepts:**

**Some students may be ready to:**

- Represent and solve multi-step problems involving positive and negative rational numbers with tape diagrams, double number lines, equations and expressions.
**Knowledge:**  *Students will know...*
- Standard algorithms for addition, subtraction, multiplication and division of multi-digit decimals

**Skills:**  *Students will be able to ...*
- Compute quotients of fractions divided by fractions. (6.NS.1)
- Explain the meaning of a quotient determined by division of fractions, using visual fraction models, equations, real-life situations, and language. (6.NS.1)
- Divide multi-digit numbers fluently using the standard algorithm. (6.NS.2)
- Fluently add, subtract, multiply and divide decimals to solve problems. (6.NS.3)

**Academic Vocabulary:**

**Critical Terms:**
- Reciprocal
- Inverse operation
- Nets
- Surface area
- Compose
- Decompose

**Supplemental Terms:**
- Quotient
- Dividend
- Divisor
- Remainder
Connections to Previous Learning:
The study of ratios and proportional relationships extends students’ work in measurement and in multiplication and division from the elementary grades. It is expected that students will have prior knowledge and experience related to concepts and skills such as multiples, factors, and divisibility rules. This background knowledge about relationships and rules for multiplication and division of whole numbers connects to the understanding of how to complete tables to help support the development of ratio and rate reasoning.

Focus of this Unit:
Students learn that a ratio expresses the comparison between two quantities. Special types of ratios are rates, unit rates, measurement conversions, and percentages are concepts that are applied to a variety of real world and mathematical situations. Students gain a deeper understanding of proportional reasoning through instruction and practice. They develop and use multiplicative thinking to develop a sense of proportional reasoning as they describe ratio relationships between two quantities.

Connections to Subsequent Learning:
Ratios and proportional relationships are foundational for further study in mathematics and science and useful in everyday life. Students use ratios in geometry and in algebra when they study similar figures and slopes of lines, and later when they study sine, cosine, tangent, and other trigonometric ratios in high school. Students use ratios when they work with situations involving constant rates of change, and later in calculus when they work with average and instantaneous rates of change of functions. An understanding of ratio is essential in the sciences to make sense of quantities that involve derived attributes such as speed, acceleration, density, surface tension, electric or magnetic field strength, and to understand percentages and ratios used in describing chemical solutions. Ratios and percentages are also useful in many situations in daily life, such as in cooking and in calculating tips, miles per gallon, taxes, and discounts. They also are also involved in a variety of descriptive statistics, including demographic, economic, medical, meteorological, and agricultural statistics (e.g., birth rate, per capita income, body mass index, rain fall, and crop yield) and underlie a variety of measures, for example, in finance (exchange rate), medicine (dose for a given body weight), and technology (kilobits per second).

From the 6-7, Ratios and Proportional Relationships Progression Document, pp. 5-7:
Representing and reasoning about ratios and collections of equivalent ratios: Because the multiplication table is familiar to sixth graders, situations that give rise to columns or rows of a multiplication table can provide good initial contexts when ratios and proportional relationships are introduced. Pairs of quantities in equivalent ratios arising from whole number measurements such as “3 lemons for every $1” or “for every 5 cups grape juice, mix in 2 cups peach juice” lend themselves to being recorded in a table. Initially, when students make tables of quantities in equivalent ratios, they may focus only on iterating the related quantities by repeated addition to generate equivalent ratios.

As students work with tables of quantities in equivalent ratios (also called ratio tables), they should practice using and understanding ratio and rate language. It is important for students to focus on the meaning of the terms “for every,” “for each,” “for each 1,” and “per” because these equivalent ways of stating ratios and rates are at the heart of understanding the structure in these tables, providing a foundation for learning about proportional relationships in Grade 7.
Students graph the pairs of values displayed in ratio tables on coordinate axes. The graph of such a collection of equivalent ratios lies on a line through the origin, and the pattern of increases in the table can be seen in the graph as coordinated horizontal and vertical increases.

By reasoning about ratio tables to compare ratios, students can deepen their understanding of what a ratio describes in a context and what quantities in equivalent ratios have in common. For example, suppose Abby’s orange paint is made by mixing 1 cup red paint for every 3 cups yellow paint and Zack’s orange paint is made by mixing 3 cups red for every 5 cups yellow. Students could discuss that all the mixtures within a single ratio table for one of the paint mixtures are the same shade of orange because they are multiple batches of the same mixture. For example, 2 cups red and 6 cups yellow is two batches of 1 cup red and 3 cups yellow; each batch is the same color, so when the two batches are combined, the shade of orange doesn’t change. Therefore, to compare the colors of the two paint mixtures, any entry within a ratio table for one mixture can be compared with any entry from the ratio table for the other mixture.

It is important for students to focus on the rows (or columns) of a ratio table as multiples of each other. If this is not done, a common error when working with ratios is to make additive comparisons. For example, students may think incorrectly that the ratios 1:3 and 3:5 of red to yellow in Abby’s and Zack’s paints are equivalent because the difference between the number of cups of red and yellow in both paints is the same, or because Zack’s paint could be made from Abby’s by adding 2 cups red and 2 cups yellow. The margin shows several ways students could reason correctly to compare the paint mixtures.

**Strategies for solving problems:** Although it is traditional to move students quickly to solving proportions by setting up an equation, the Standards do not require this method in Grade 6. There are a number of strategies for solving problems that involve ratios. As students become familiar with relationships among equivalent ratios, their strategies become increasingly abbreviated and efficient. For example, suppose grape juice and peach juice are mixed in a ratio of 5 to 2 and we want to know how many cups of grape juice to mix with 12 cups of peach juice so that the mixture will still be in the same ratio. Students could make a ratio table as shown in the margin, and they could use the table to find the grape juice entry that pairs with 12 cups of peach juice in the table. This perspective allows students to begin to reason about proportions by starting with their knowledge about multiplication tables and by building on this knowledge.
As students generate equivalent ratios and record them in tables, their attention should be drawn to the important role of multiplication and division in how entries are related to each other. Initially, students may fill ratio tables with columns or rows of the multiplication table by skip counting, using only whole number entries, and placing these entries in numerical order. Gradually, students should consider entries in ratio tables beyond those they find by skip counting, including larger entries and fraction or decimal entries. Finding these other entries will require the explicit use of multiplication and division, not just repeated addition or skip counting. For example, if Seth runs 5 meters every 2 seconds, then Seth will run 2.5 meters in 1 second because in half the time he will go half as far. In other words, when the elapsed time is divided by 2, the distance traveled should also be divided by 2. More generally, if the elapsed time is multiplied (or divided) by \(N\), the distance traveled should also be multiplied (or divided) by \(N\). Double number lines can be useful in representing ratios that involve fractions and decimals.

As students become comfortable with fractional and decimal entries in tables of quantities in equivalent ratios, they should learn to appreciate that unit rates are especially useful for finding entries. A unit rate gives the number of units of one quantity per 1 unit of the other quantity. The amount for \(N\) units of the other quantity is then found by multiplying by \(N\). Once students feel comfortable doing so, they may wish to work with abbreviated tables instead of working with long tables that have many values. The most abbreviated tables consist of only two columns or two rows; solving a proportion is a matter of finding one unknown entry in the table.
Measurement conversion provides other opportunities for students to use relationships given by unit rates. For example, recognizing “12 inches in a foot,” “1000 grams in a kilogram,” or “one kilometer is $\frac{5}{8}$ of a mile” as rates, can help to connect concepts and methods developed for other contexts with measurement conversion.

**Representing a problem with a tape diagram**

Slimy Gloopy mixture is made by mixing glue and liquid laundry starch in a ratio of 3 to 2. How much glue and how much starch is needed to make 95 cups of Slimy Gloopy mixture?

- 5 parts $\rightarrow$ 85 cups
- 1 part $\rightarrow$ $\frac{85}{5} = 17$ cups
- 3 parts $\rightarrow$ 3 x 17 = 51 cups
- 2 parts $\rightarrow$ 2 x 17 = 34 cups

51 cups glue and 34 cups starch are needed.

Tape diagrams can be useful aids for solving problems.

**Representing a multi-step problem with two pairs of tape diagrams**

Yellow and blue paint were mixed in a ratio of 5 to 3 to make green paint. After 14 liters of blue paint were added, the amount of yellow and blue paint in the mixture was equal. How much green paint was in the mixture at first?

At first:
- Yellow: $\frac{5}{8}$ of 14 liters
- Blue: $\frac{3}{8}$ of 14 liters

Then:
- Yellow: $\frac{3}{8}$ of 14 liters
- Blue: $\frac{5}{8}$ of 14 liters

2 parts $\rightarrow$ 14 liters
1 part $\rightarrow$ $\frac{14}{2} = 7$ liters
(original total) 8 parts $\rightarrow$ $8 \times 7 = 56$ liters

There was 56 liters of green paint to start with.

This problem can be very challenging for sixth or seventh graders.
### Desired Outcomes

#### Standard(s):

**Understand ratio concepts and use ratio reasoning to solve problems.**

1. **6.RP.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”*

2. **6.RP.2** Understand the concept of a unit rate \( \frac{a}{b} \) associated with a ratio \( \frac{a:b} \) with \( b \neq 0 \), and use rate language in the context of a ratio relationship. *For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is \( \frac{3}{4} \) cup of flour for each cup of sugar.” “We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.”*

3. **6.RP.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
   a) Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
   b) Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
   c) Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
   d) Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

#### Represent and analyze quantitative relationships between dependent and independent variables.

4. **6.EE.9** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation \( d = 65t \) to represent the relationship between distance and time.*

### WIDA Standard: (English Language Learners)

English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics. English Language Learners benefit from:

- practice with manipulatives (such as fraction-decimal-percent equivalence towers, fraction squares for multiplication and division, etc.) and visuals (such as tape diagrams).
- explicit vocabulary instruction to connect the content to language.
### Understandings:
*Students will understand that …*
- A ratio expresses the comparison between two quantities. Special types of ratios are rates, unit rates, measurement conversions, and percents.
- A ratio or a rate expresses the relationship between two quantities. Ratio and rate language is used to describe a relationship between two quantities (including unit rates.)
- A rate is a type of ratio that represents a measure, quantity, or frequency, typically one measured against a different type of measure, quantity, or frequency.
- Ratio and rate reasoning can be applied to many different types of mathematical and real-life problems (rate and unit rate problems, scaling, unit pricing, statistical analysis, etc.).

### Essential Questions:
- When is it useful to be able to relate one quantity to another?
- How are ratios and rates similar and different?
- What is the connection between a ratio and a fraction?

### Mathematical Practices: (Practices to be explicitly emphasized are indicated with an *.)
1. **Make sense of problems and persevere in solving them.** Students understand the problem context in order to translate them into ratios/rates.
2. **Reason abstractly and quantitatively.** Students understand the relationship between two quantities in order to express them mathematically. They use ratio and rate notation as well as visual models and contexts to demonstrate reasoning.
3. **Construct viable arguments and critique the reasoning of others.** Students construct and critique arguments regarding appropriateness of representations given ratio and rate contexts. For example, does a tape diagram adequately represent a given ratio scenario.
4. **Model with mathematics.** Students can model problem situations symbolically (tables, expressions or equations), visually (graphs or diagrams) and contextually to form real-world connections.
5. **Use appropriate tools strategically.** Students choose appropriate models for a given situation, including tables, expressions or equations, tape diagrams, number line models, etc.
6. **Attend to precision.** Students use and interpret mathematical language to make sense of ratios and rates.
7. **Look for and make use of structure.** The structure of a ratio is unique and can be used across a wide variety of problem-solving situations. For instance, students recognize patterns that exist in ratio tables, including both the additive and multiplicative properties. In addition, students use their knowledge of the structures of word problems to make sense of real-world problems.
8. **Look for and express regularity in repeated reasoning.** Students utilize repeated reasoning by applying their knowledge of ratio, rate and problem solving structures to new contexts. Students can generalize the relationship between representations, understanding that all formats represent the same ratio or rate.
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<thead>
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<th>Knowledge:</th>
<th>Students will know...</th>
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<tr>
<td><strong>Prerequisite Skills/Concepts:</strong></td>
<td><strong>Advanced Skills/Concepts:</strong></td>
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</tbody>
</table>
| *Students should already understand:* | *Some students may be ready to:*
| • Multiples and Factors | Students will use ratios, rates, unit rates and percent skills: |
| • Divisibility Rules | • in grade 7 when working with proportional relationships and probability |
| • Relationships and rules for multiplication and division of whole numbers as they apply to decimal fractions | • in geometry and in algebra when studying similar figures and slopes of lines |
| • Understanding of common fractions | **Skills:** Students will be able to... |
| **Knowledge:** | **Skills:** Students will be able to... |
| • A **ratio** compares two related quantities. | • Use ratio language to describe a ratio relationship between two quantities. (6.RP.1) |
| • Ratios can be represented in a variety of formats including each, to, per, for each, %, 1/5, etc. | • Represent a ratio relationship between two quantities using manipulatives and/or pictures, symbols and real-life situations. \( \text{a to b, } \frac{a}{b}, \text{ or } a:b \) (6.RP.1) |
| • A **percent** is a type of ratio that compares a quantity to 100. | • Represent unit rate associated with ratios using visuals, charts, symbols, real-life situations and rate language. (6.RP.2) |
| • A **unit rate** is the ratio of two measurements in which the second term is 1. | • Use ratio and rate reasoning to solve real-world and mathematical problems. (6.RP.3) |
| • When it is appropriate to use ratios/rates to solve mathematical or real life problems. | • Make and interpret tables of equivalent ratios. (6.RP.3) |
| • Mathematical strategies for solving problems involving ratios and rates, including tables, tape diagrams, double line diagrams, equations, equivalent fractions, graphs, etc | • Plot pairs of values of the quantities being compared on the coordinate plane. (6.RP.3) |
| **Skills:** | **Skills:** |
| Students will know... | Students will be able to... |
| • Use ratio language to describe a ratio relationship between two quantities. (6.RP.1) | • Use multiple representations such as tape diagrams, double number line diagrams, or equations to solve rate and ratio problems. (6.RP.3) |
| • Represent a ratio relationship between two quantities using manipulatives and/or pictures, symbols and real-life situations. (6.RP.1) | • Solve unit rate problems (including unit pricing and constant speed). (6.RP.3) |
| • Represent unit rate associated with ratios using visuals, charts, symbols, real-life situations and rate language. (6.RP.2) | • Solve percent problems, including finding a percent of a quantity as a rate per 100 and finding the whole, given the part and the percent. (6.RP.3) |
| • Use ratio and rate reasoning to solve real-world and mathematical problems. (6.RP.3) | • Use variables to represent two quantities in a real-world problem that change in relationship to one another. (6.EE.9) |
| • Make and interpret tables of equivalent ratios. (6.RP.3) | • Write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. (6.EE.9) |
| • Plot pairs of values of the quantities being compared on the coordinate plane. (6.RP.3) | • Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. (6.EE.9) |

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Adapted from UbD framework

**Priority Standards** = Approximately 70%

**Supporting Standards** = Approximately 20%

**Additional Standards** = Approximately 10%
### Grade 6: Unit 2: Ratios, Rates and Proportions

#### Academic Vocabulary:

<table>
<thead>
<tr>
<th>Critical Terms:</th>
<th>Supplemental Terms:</th>
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<tbody>
<tr>
<td>Percent</td>
<td>Tape diagram</td>
</tr>
<tr>
<td>Proportion</td>
<td>Double number line</td>
</tr>
<tr>
<td>Rate</td>
<td>Numerator</td>
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<tr>
<td>Ratio</td>
<td>Denominator</td>
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<tr>
<td>Rational number</td>
<td>Equivalent</td>
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<tr>
<td>Unit Ratio</td>
<td></td>
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<tr>
<td>Quantity</td>
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Connections to Previous Learning:
Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. Examples that use positive and negative numbers to describe nature, financial credits and debits, or electricity help build a context for learning about integers and the meaning of “0”.

Focus of the Unit:
Much of the learning in this unit is related to distances on a number line. Students learn that between two whole numbers on a number line, there are points that are described by rational numbers. Students compare and order rational numbers on the number line using statements about the relative position of the numbers on the line and record these comparisons using inequalities. For instance, -5 > -8 is described as -5 is located to the right of -8 on a number line oriented from left to right. Nature, finances or temperatures might be used as contexts to describe the numbers. For instance, -3° Centigrade is warmer than -7°.

Students’ experiences placing rational numbers on vertical and horizontal number lines prepare them to plot points in all 4 quadrants of the coordinate plane. They see the sign of the number as an indicator of directionality and the number, itself as the distance a point is from zero, or the origin. They reason about the order and absolute value of rational numbers, and learn to interpret absolute value |5| as the magnitude for a negative or positive number. For example, for a money account of – 5 dollars, the |5| means the quantity of money owed or debited.

Through experiences with number lines and other contexts, students learn that the opposite of the opposite of a number is the number itself, e.g., - (-6) = 6 and they learn that 0 is its own opposite.

Connections to Subsequent Learning:
Students in Grade 6 also build on their work with distance in elementary school by reasoning about relationships among shapes to determine distances. Students will apply what they learn about integers to their work on expressions in Unit 4. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.
Standard(s):

Apply and extend previous understandings of numbers to the system of rational numbers.

- **6.NS.5** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

- **6.NS.6** Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
  a) Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., -(-3) = 3, and that 0 is its own opposite.
  b) Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
  c) Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

- **6.NS.7** Understand ordering and absolute value of rational numbers.
  a) Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret – 3 > - 7 as a statement that – 3 is located to the right of -7 on a number line oriented from left to right.
  b) Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write -3°C > -7°C to express the fact that -3°C is warmer than -7°C.
  c) Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of – 30 dollars, write │-30│ = 30 to describe the size of the debt in dollars.
  d) Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than – 30 dollars represents a debt greater than 30 dollars.

- **6.NS.8** Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Solve real-world and mathematical problems involving area, surface area, and volume.

- **6.G.3** Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
WIDA Standard: (English Language Learners)
English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.

English language learners benefit from:

- the use of visuals to describe the contexts of positive and negative number situations.
- awareness that the number line going from smaller numbers on the left to a larger number to the right is similar to reading from left to right. Students whose languages read in different directions may need more explicit practice to master this work using the number line.

Understandings: **Students will understand that ...**

- Quantities having more or less than zero are described using positive and negative numbers.
- Number lines are visual models used to represent the density principle: between any two whole numbers are many rational numbers, including decimals and fractions.
- The rational numbers can extend to the left or to the right on the number line, with negative numbers going to the left of zero, and positive numbers going to the right of zero.
- The coordinate plane is a tool for modeling real-world and mathematical situations and for solving problems.

Essential Questions:

- How are positive and negative numbers used?
- How do rational numbers relate to integers?
- What is modeled on the coordinate plane?
**Mathematical Practices:** (Practices to be explicitly emphasized are indicated with an *.)

1. **Make sense of problems and persevere in solving them.** Students make sense of problems involving points and polygons in the coordinate plane.

2. **Reason abstractly and quantitatively.** Students demonstrate abstract reasoning about rational numbers with their visual representations. Students consider the values of these numbers in relation to distance (number lines).

3. **Construct viable arguments and critique the reasoning of others.** Students construct and critiques arguments regarding number line representations and the use of inequalities to represent real-world contexts.

4. **Model with mathematics.** Students use number lines to compare numbers and represent inequalities in mathematical and real-world contexts.

5. **Use appropriate tools strategically.** Students select and use tools such as two-color counters, number line models and the coordinate plane to represent situations involving positive and negative numbers.

6. **Attend to precision.** Students attend to the language of real-world situations to determine if positive or negative quantities/distances are being represented.

7. **Look for and make use of structure.** Students relate the structure of number lines to values of rational numbers as they use the coordinate plane.

8. **Look for and express regularity in repeated reasoning.** Students relate new experiences to experiences with similar contexts when studying positive and negative representations of distance and quantity. In the study of absolute value, students demonstrate repeated reasoning by showing that both positive and negative quantities represent the same distance from zero.

**Prerequisite Skills/Concepts:**

**Students should already be able to:**
- Represent positive rational numbers on a number line and compare values of these numbers.
- Plot points on the coordinate plane and connect the visual representation to real-life situations, oral/written language, and tables.

**Advanced Skills/Concepts:**

**Some students may be ready to:**
- Use coordinates and absolute value to find distances between points where the first coordinate or the second coordinate are not the same.
- Create transformations, such as translations, rotations and reflections based on coordinate shifts.

**Knowledge:** *Students will know*...

All standards for this unit go beyond the knowledge level.

**Skills:** *Students will be able to* ...

- Identify an integer and its opposite and the directions they represent in real-world contexts. (6.NS.5)
- Use integers to represent quantities in real-world situations (above/ below sea level) (6.NS.5)
- Understand the meaning of 0 and where it fits into a situation(6.NS.5)
- Represent and explain the value of a rational number as a point on a number line (6.NS.6)
- Recognize that a number line can be both vertical and horizontal (6.NS.6)
- Represent a number and its opposite equidistant from zero on a number line. (6.NS.6)
- Identify that the opposite of the opposite of the number is itself. (6.NS.6)
- Incorporate opposites on the number line or plot opposite points on a coordinate grid where x and y intersect at zero. (6.NS.6)
- Represent signs of numbers in ordered pairs as locations in quadrants on the coordinate plane and explain the relationship between the location and the signs. (6.NS.6)
- Represent and explain reflections of ordered pairs on a coordinate plane. (6.NS.6)
- Locate and position integers and other rational numbers on horizontal or vertical number lines (6.NS.6)
- Locate and position integers and other rational numbers on a coordinate plane. (6.NS.6)
- Identify the absolute value of a number as the distance from zero (6.NS.7)
- Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. (6.NS.7)
- Use inequalities to order integers relative to their position on the number line. (6.NS.7)
- Write statements of order for rational numbers in real-world contexts. (6.NS.7)
- Interpret statements of order for rational numbers in real-world contexts. (6.NS.7)
- Explain statements of order for rational numbers in real-world contexts. (6.NS.7)
- Represent the absolute value of a rational number as the distance from zero and recognize the symbol | x |. (6.NS.7)
- Interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. (6.NS.7)
- Distinguish comparisons of absolute value from statements about order. (Compare rational numbers using absolute value in real-world situations. For negative numbers, as the absolute values increases, the value of the number decreases.) (6.NS.7)
- Solve real-world problems by graphing points in all four quadrants of the coordinate plane. (6.NS.8)
- Use coordinates to find distances between points with the same first coordinate or the same second coordinate. (6.NS.8)
- Use absolute value to find distances between points with the same first coordinate or the same second coordinate. (6.NS.8)
- Draw polygons in the coordinate plane given the coordinates for the vertices. (6.G.3)
- Use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. (6.G.3)
- Solve real-world and mathematical problems involving polygons in the coordinate plane. (6.G.3)
### Academic Vocabulary:

<table>
<thead>
<tr>
<th>Critical Terms:</th>
<th>Supplemental Terms:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Integers</td>
<td>Coordinate</td>
</tr>
<tr>
<td>Rational numbers</td>
<td>Ordered pairs</td>
</tr>
<tr>
<td>Quadrants</td>
<td>Input</td>
</tr>
<tr>
<td>Line diagrams</td>
<td>Output</td>
</tr>
<tr>
<td>Absolute value</td>
<td>x-coordinate</td>
</tr>
<tr>
<td>Positive</td>
<td>y-coordinate</td>
</tr>
<tr>
<td>Negative</td>
<td>x-axis</td>
</tr>
<tr>
<td>Opposite</td>
<td>y-axis</td>
</tr>
<tr>
<td></td>
<td>origin</td>
</tr>
<tr>
<td></td>
<td>distance</td>
</tr>
</tbody>
</table>

- **Priority Standards** = Approximately 70%
- **Supporting Standards** = Approximately 20%
- **Additional Standards** = Approximately 10%
Connections to Previous Learning:
Students have been writing numerical expressions since Kindergarten, such as 2 + 3. In grade 5 they used whole number exponents to express powers of 10, and in Grade 6 they start to incorporate whole number exponents into numerical expressions. Students have also been using letters to represent an unknown quantity in word problems since Grade 3. In Grade 6 they begin to work systematically with algebraic expressions. Students in Grade 5 began to move from viewing expressions as actions describing a calculation to viewing them as objects in their own right. They describe the structure of expressions, seeing them as a product of two factors, the second of which can be viewed as both a single entity and a sum of two terms.

Focus of the Unit:
In this unit, students understand the use of variables in mathematical expressions. They become more fluent at viewing expressions as objects in their own right versus calculations. Students write expressions that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students will understand that expressions in different forms can be equivalent, and they will use the properties of operations to generate and rewrite expressions in equivalent forms. The Mathematical Practices should be evident throughout instruction of symbolic expressions and connected to the content. Students should engage in mathematical tasks that provide an opportunity to connect content and practices.

Connections to Subsequent Learning:
Students extend their previous understandings of numerical expressions. They continue to develop their understanding of how letters are used to represent numbers in mathematics, in both equality and inequality statements. They will use tables, words, numbers, graphs, and equations to describe relationships. Students will solve complex problems in problem solving situations using expressions including the use of the idea of dependent and independent variables.

From the 6-8, Expressions and Equations Progression Document – Overview p. 2:
An expression expresses something. Facial expressions express emotions. Mathematical expressions express calculations with numbers. Some of the numbers might be given explicitly, like 2 or \( \frac{3}{4} \). Other numbers in the expression might be represented by letters, such as \( x \), \( y \), \( P \), or \( n \). The calculation an expression represents might use only a single operation, as in \( 4 + 3 \) or \( 3x \), or it might use a series of nested or parallel operations, as in \( 3(a + 9) - 9/b \). An expression can consist of just a single number, even 0.

Letters standing for numbers in an expression are called variables. In good practice, including in student writing, the meaning of a variable is specified by the surrounding text; an expression by itself gives no intrinsic meaning to the variables in it. Depending on the context, a variable might stand for a specific number, for example the solution to a word problem; it might be used in a universal statement true for all numbers, for example when we say that that \( a + b = b + a \) for all numbers \( a \) and \( b \); or it might stand for a range of numbers, for example when we say that \( \sqrt{x^2} - x \) for \( x > 0 \). In choosing variables to represent quantities, students specify a unit; rather than saying “let \( G \) be gasoline,” they say “let \( G \) be the number of gallons of gasoline”.

An expression is a phrase in a sentence about a mathematical or real-world situation. As with a facial expression, however, you can read a lot from an algebraic expression (an expression with variables in it) without knowing the story behind it, and it is a goal of this progression for students to see expressions as objects in their own right, and to read the general appearance and fine details of algebraic expressions.

An equation is a statement that two expressions are equal, such as \( 10 + 0.02n = 20 \), or \( 3 + x - 4 + x \), or \( 2(a + 1) = 2a + 2 \). An important aspect of equations that the two expressions on either side of the equal sign might not actually always be equal; that is, the equation might be a true statement for some values of the variable(s) and a false statement for others. For example, \( 10 + 0.2n = 20 \) is true only if \( n = 500 \); and \( 3 + x = 4 + x \) is not true for any number \( x \);
and $2(a + 1) = 2a + 2$ is true for all numbers $a$. A solution to an equation is a number that makes the equation true when substituted for the variable (or, if there is more than one variable, it is a number for each variable). An equation may have no solutions (e.g., $3 + x = 4 + x$) has no solutions because, no matter what number $x$ is, it is not true that adding 3 to $x$ yields the same answer as adding 4 to $x$). An equation may also have every number for a solution (e.g., $2p - 1q = 2a$). An equation that is true no matter what number the variable represents is called an identity, and the expressions on each side of the equation are said to be equivalent expressions. For example $2(a + 1)$ and $2a + 2$ are equivalent expressions. In Grades 6–8, students start to use properties of operations to manipulate algebraic expressions and produce different but equivalent expressions for different purposes. This work builds on their extensive experience in K–5 working with the properties of operations in the context of operations with whole numbers, decimals and fractions.

From the 6-8, Expressions and Equations Progressions Document pp. 4-6:

**Apply and extend previous understandings of arithmetic to algebraic expressions:** Students have been writing numerical expressions since Kindergarten, such as:

$2 + 3 \quad 7 + 6 + 3 \quad 4 \times (2 \times 3)$

$8 \times 5 + 8 \times 2 \quad \frac{1}{3} (8 + 7 + 3) \quad \frac{3}{2}$

In Grade 5 they used whole number exponents to express powers of 10, and in Grade 6 they start to incorporate whole number exponents into numerical expressions, for example when they describe a square with side length 50 feet as having an area of 50$^2$ square feet.

Students have also been using letters to represent an unknown quantity in word problems since Grade 3. In Grade 6 they begin to work systematically with algebraic expressions. They express the calculation “Subtract $y$ from 5” as $5 - y$, and write expressions for repeated numerical calculations. For example, students might be asked to write a numerical expression for the change from a $10 bill after buying a book at various prices:

<table>
<thead>
<tr>
<th>Price of book ($)</th>
<th>5.00</th>
<th>6.49</th>
<th>7.15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change from $10</td>
<td>10-5</td>
<td>10-6.49</td>
<td>10-7.15</td>
</tr>
</tbody>
</table>

Abstracting the pattern they write $10 - p$ for a book costing $p$ dollars, thus summarizing a calculation that can be carried out repeatedly with different numbers. Such work also helps students interpret expressions. For example, if there are 3 loose apples and 2 bags of $A$ apples each, students relate quantities in the situation to the terms in the expression $3 + 2A$.

As they start to solve word problems algebraically, students also use more complex expressions. For example, in solving the word problem

Daniel went to visit his grandmother, who gave him $5.50. Then he bought a book costing $9.20. If he has $2.30 left, how much money did he have before visiting his grandmother?

students might obtain the expression $x + 5.50 - 9.20$ by following the story forward, and then solve the equation $x + 5.50 - 9.20 = 2.30$. Students may need explicit guidance in order to develop the strategy of working forwards, rather than working backwards from the 2:30 and calculating $2.30 + 9.20 - 5.50$. As word problems get more complex, students find greater benefit in representing the problem algebraically by choosing variables to represent quantities, rather than attempting a direct numerical solution, since the former approach provides general methods and relieves demands on working memory.
Students in Grade 5 began to move from viewing expressions as actions describing a calculation to viewing them as objects in their own right; in Grade 6 this work continues and becomes more sophisticated. They describe the structure of an expression, seeing \(2(8 + 7)\) for example as a product of two factors the second of which, \((8 + 7)\), can be viewed as both a single entity and a sum of two terms. They interpret the structure of an expression in terms of a context: if a runner is \(7t\) miles from her starting point after \(t\) hours, what is the meaning of the \(7\)? If \(a\), \(b\), and \(c\) are the heights of three students in inches, they recognize that the coefficient \(\frac{1}{3}\) in \(\frac{1}{3}(a + b + c)\) has the effect of reducing the size of the sum, and they also interpret multiplying by \(\frac{1}{3}\) as dividing by 3. Both interpretations are useful in connection with understanding the expression as the mean of \(a\), \(b\), and \(c\).

In the work on number and operations in Grades K–5, students have been using properties of operations to write expressions in different ways. For example, students in grades K–5 write \(2 + 3 = 3 + 2\) and \(8 \times 5 + 8 \times 2 = 8 \times (5 + 2)\) and recognize these as instances of general properties which they can describe. They use the “any order, any grouping” property to see the expression \(7 + 6 + 3\) as \((7 + 3) + 6\), allowing them to quickly evaluate it. The properties are powerful tools that students use to accomplish what they want when working with expressions and equations. They can be used at any time, in any order, whenever they serve a purpose. Work with numerical expressions prepares students for work with algebraic expressions. During the transition, it can be helpful for them to solve numerical problems in which it is more efficient to hold numerical expressions unevaluated at intermediate steps. For example, the problem

Fred and George Weasley make 150 “Deflagration Deluxe” boxes of Weasleys’ Wildfire Whiz-bangs at a cost of 17 Galleons each, and sell them for 20 Galleons each. What is their profit?

Is more easily solved by unevaluated the total cost, \(150 \times 17\) Galleons, and the total revenue \(150 \times 20\) Galleons, until the subtraction step, where the distributive law can be used to calculate the answer as \(150 \times 20 - 150 \times 17 = 150 \times 3 = 450\) Galleons. A later algebraic version of the problem might ask for the sale price that will yield a given profit, with the sale price represented by a letter such as \(p\). The habit of leaving numerical expressions unevaluated prepares students for constructing the appropriate algebraic equation to solve such a problem.

As students move from numerical to algebraic work the multiplication and division symbols \(\times\) and \(\div\) are replaced by the conventions of algebraic notation. Students learn to use either a dot for multiplication, e.g., \(1 \cdot 2 \cdot 3\) instead of \(1 \times 2 \times 3\), or simple juxtaposition, e.g., \(3x\) instead of \(3 \times x\) (during the transition, students may indicate all multiplications with a dot, writing \(3 \cdot x\) for \(3x\)). A firm grasp on variables as numbers helps students extend their work with the properties of operations from arithmetic to algebra. For example, students who are accustomed to mentally calculating \(5 \times 37\) as \(5 \times (30 + 7) = 150 + 35\) can now see that \(5(3a + 7) = 15a + 35\) for all numbers \(a\). They apply the distributive property to the expression \(3(2 + x)\) to produce the equivalent expression \(6 + 3x\) and to the expression \(24x + 18y\) to produce the equivalent expression \(6(4x + 3y)\).

Students evaluate expressions that arise from formulas used in real-world problems, such as the formulas \(V = s^3\) and \(A = 6s^2\) for the volume and surface area of a cube. In addition to using the properties of operations, students use conventions about the order in which arithmetic operations are performed in the absence of parentheses. It is important to distinguish between such conventions, which are notational conveniences that allow for algebraic expressions to be written with fewer parentheses, and properties of operations, which are fundamental properties of the number system and undergird all work with expressions. In particular, the mnemonic PEMDAS can mislead students into thinking, for example, that addition must always take precedence over subtraction because the A...
comes before the S, rather than the correct convention that addition and subtraction proceed from left to right (as do multiplication and division). This can lead students to make mistakes such as simplifying $n - 2 + 5$ as $n - 7$ (instead of the correct $n + 3$) because they add the 2 and the 5 before subtracting from $n$.

The order of operations tells us how to interpret expressions, but does not necessarily dictate how to calculate them. For example, the P in PEMDAS indicates that the expression $8 \times (5 + 1)$ is to be interpreted as 8 times a number which is the sum of 5 and 1. However, it does not dictate the expression must be calculated this way. A student might well see it, through an implicit use of the distributive law, as $8 \times 5 + 8 \times 1 = 40 + 8 = 48$.

The distributive law is of fundamental importance. Collecting like terms, e.g., $5b + 3b = (5 + 3)b - 8b$, should be seen as an application of the distributive law, not as a separate method.

### Desired Outcomes

**Standard(s):**

Apply previous understandings of arithmetic to algebraic expressions

- **6.EE.1** Write and evaluate numerical expressions involving whole-number exponents.
- **6.EE.2** Write, read and evaluate expressions in which letters stand for numbers.
  - a) Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation “Subtract y from 5” as $5 - y$.*
  - b) Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entry. *For example, describe the expression $2 (8 + 7)$ as a product of two factors; view $(8 + 7)$ as both a single entity and a sum of two terms.*
  - c) Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = S^3$ and $A = 6S^2$ to find the volume and surface area of a cube with sides of lengths $s = 1/2$.*
- **6.EE.3** Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression $3 (2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6 (4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$.
- **6.EE.4** Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions $y + y + y$ and $3y$ are equivalent because the name the same number regardless of which number $y$ stands for.*

**Compare fluently with multi-digit numbers and find common factors and multiples.**

- **6.NS.4** Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express $36 + 8$ as $4 (9 + 2)$.*
WIDA Standard: (English Language Learners)
English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.
ELLs will benefit from
- explicit instruction in the transfer between verbal descriptions and algebraic expressions.
- explicit examples of mathematical terms: sum, term, product, factor, quotient, coefficient, etc.
- manipulatives (such as algeblocks, algebra tiles or hands-on-equations) to model strategies for evaluating expressions.

Understandings: Students will understand that ...
- Properties of operations are used to determine if expressions are equivalent.
- There is a designated sequence to perform operations (Order of Operations).
- Variables can be used as unique unknown values or as quantities that vary.
- Algebraic expressions may be used to represent and generalize mathematical problems and real life situations

Essential Questions:
- What is equivalence?
- How properties of operations used to prove equivalence?
- How are variables defined and used?

Mathematical Practices: (Practices to be explicitly emphasized are indicated with an *.)
1. Make sense of problems and persevere in solving them. Students make sense of expressions by connecting them to real world contexts when evaluating.
2. Reason abstractly and quantitatively. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
3. Construct viable arguments and critique the reasoning of others. Students construct and critique arguments regarding the equivalence of expressions and the use of variable expressions to represent real-world situations.
4. Model with mathematics. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations.
5. Use appropriate tools strategically. Students determine which algebraic representations are appropriate for given contexts.
6. Attend to precision. Students use the language of real-world situations to create appropriate expressions.
7. Look for and make use of structure. Students apply properties to generate equivalent expressions. They interpret the structure of an expression in terms of a context.
8. Look for and express regularity in repeated reasoning. Students can work with expressions involving variables without the focus on a specific number or numbers that the variable may represent they can focus on the patterns that occur. It is these patterns that lead to generalizations that lay the foundation for their future work in algebra.
### Prerequisite Skills/Concepts:

**Students should already be able to:**
- Define a variable.
- Identify and differentiate between common factors and common multiples of two whole numbers.

### Advanced Skills/Concepts:

**Some students may be ready to:**
- Understand that the properties of operations hold for integers, rational, and real numbers.
- Use the properties of operations to rewrite equivalent numerical expressions using non-negative rational numbers.
- Use variables to represent real-world situations and use the properties of operations to generate equivalent expressions for these situations.
- Experience expressions for amounts of increase and decrease.
- Use substitution to understand that expressions are equivalent.
- Solve complex problems involving expressions.

### Knowledge:  *Students will know...*

- Exponential notation is a way to express repeated products of the same number.

### Skills:  *Students will be able to ...*

- Write numerical expressions that have whole number exponents. (6.EE.1)
- Evaluate numerical expressions that have whole number exponents and rational bases. (6.EE.1)
- Write algebraic expressions to represent real life and mathematical situations (6.EE.2)
- Identify parts of an expression using appropriate terminology (6.EE.2)
- Given the value of a variable, students will evaluate the expression (6.EE.2)
- Use order of operations to evaluate expressions. (6.EE.2)
- Apply properties of operations to write equivalent expressions. (6.EE.3)
- Identify when two expressions are equivalent. (6.EE.4)
- Prove (using various strategies) that two equations are equivalent no matter what number is substituted. (6.EE.4)
- Identify the factors of any whole number less than or equal to 100. (6.NS.4)
- Determine the Greatest Common Factor of two or more whole numbers less than or equal to 100. (6.NS.4)
- Identify the multiples of two whole numbers less than or equal to 12 and determine the Least Common Multiple. (6.NS.4)
- Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. (6.NS.4)
### Grade 6 Unit 4 Expressions

<table>
<thead>
<tr>
<th>Academic Vocabulary:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Critical Terms:</strong></td>
<td><strong>Supplemental Terms:</strong></td>
</tr>
<tr>
<td>superscripted numbers</td>
<td>Dividend</td>
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<tr>
<td>equivalent</td>
<td>Divisor</td>
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<td>Equation</td>
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<td>Algebraic expression</td>
<td>identity property</td>
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<td>Base</td>
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<tr>
<td>Term</td>
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</tr>
<tr>
<td>greatest common factor (GCF)</td>
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<tr>
<td>least common multiple (LCM)</td>
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<tr>
<td>prime factorization</td>
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</tbody>
</table>

**Priority Standards** = Approximately 70%

**Supporting Standards** = Approximately 20%

**Additional Standards** = Approximately 10%
Connections to Previous Learning:
In Fifth grade, students write simple expressions to record calculations with numbers or interpret the operations of the expression. Students have experience generating a relationship between an input number, the rule and an output number. In preparation for this 6th grade unit, prior learning in sixth grade includes the procedural knowledge of how to divide a fraction by a fraction (6.NS.1) which may be required for solving equations of the form $px = q$ (6.EE.7). Recognizing the role of a letter as a variable that can represent a number in reading, writing or evaluating expressions or equations (6.EE.2) is prerequisite skill for solving equations (6.EE.5), using variables (6.EE.6), using equations to solve problems (6.EE.7) and writing inequalities (6.EE.8).

Focus of this Unit:
In this unit, understand Solving an Equation or Inequality is based on understanding the important role equivalence plays in the number and operation strand of mathematics. Based on the equivalence understanding, students learn a process for solving equations (6.EE.5), and begin to see the usefulness of variables (6.EE.8). Students learn to use equations and inequalities to describe relationships in data or in patterns of numbers or shapes, and then make statements about these relationships based on the structure of mathematics. This includes processes such as: using substitution to make an equation true, and using variables to represent numbers and inequalities. Students practice using critical thinking to solve word problems using number lines and equations to model thinking.

Connections to Subsequent Learning:
In subsequent units, students will use their understanding of solving equations and equivalence to solve problems using algebraic formulas and graphs.

From the 6-8, Expressions and Equations Progression Document pp. 6-7:
Reason about and solve one-variable equations and inequalities: In Grades K-5 students have been writing numerical equations and simple equations involving one operation with a variable. In Grade 6 they start their systematic study of equations and inequalities and methods of solving them. Solving is a process of reasoning to find the numbers which make an equation true, which can include checking if a given number is a solution. Although the process of reasoning will eventually lead to standard methods for solving equations, students should study examples where looking for structure pays off, such as in $4x + 3x = 3x + 20$, where they can see that $4x$ much be 20 to make the two sides equal.

This understanding can be reinforced by comparing arithmetic and algebraic solutions to simple word problems. For example, how many 44-cent stamps can you buy with $11? Students are accustomed to solving such problems by division; now they see the parallel with representing the problem algebraically as $0.44n - 11$, from which they use the same reasoning as in the numerical solution to conclude that $n = 11 ÷ 0.44$. They explore methods such as dividing both sides by the same non-zero number. As word problems grow more complex in Grades 6 and 7, analogous arithmetical and algebraic solutions show the connection between the procedures of solving equations and the reasoning behind these procedures. When students start studying in one variable, it is important for them to understand every occurrence of a given variable has the same value in the expression and throughout a solution procedure: if $x$ is assumed to be the number satisfying the equation $4x + 3x = 3x + 20$ at the beginning of a solution procedure, it remains that number throughout.
As with all their work with variables, it is important for students to state precisely the meaning of variables they use when setting up equations. This includes specifying whether the variable refers to a specific number, or to all numbers in some range. For example, in the equation $0.44n$ is presented as a general formula for calculating the price in dollars of $n$ stamps, and then $n$ refers to all numbers in some domain. That domain might be specified by inequalities such as $n > 0$.

### Desired Outcomes

**Standard(s):**
Reason about and solve one-variable equations and inequalities.

- **6.EE.5** Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use and explain substitution in order to determine whether a given number in a specified set makes an equation or inequality true.
- **6.EE.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
- **6.EE.7** Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which $p$, $q$ and $x$ are all nonnegative rational numbers.
- **6.EE.8** Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

**WIDA Standard: (English Language Learners)**

English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics. English language learners benefit from:

- manipulatives to aid in representing and solving equations and inequalities (such as algebra tiles, algeblocks or hands-on-equations).
- number line representations when representing and solving equations and inequalities.
- strategies for articulating the identity of variables when used in expressions, equations and inequalities that represent real-world situations.
## Understandings: *Students will understand that* ...

- Solving equations is a reasoning process and follows established procedures based on properties.
- Substitution is used to determine whether a given number in a set makes an equation or inequality true.
- Variables may be used to represent a specific number, or, in some situations, to represent all numbers in a specified set.
- When one expression has a different value than a related expression, an inequality provides a way to show that relationship between the expressions: the value of one expression is greater than (or greater than or equal to) the value of the other expression instead of being equal.
- Inequalities may have infinite solutions and there are methods for determining if an inequality has infinite solutions using graphs and equations.
- Solutions of inequalities can be represented on a number line.
- Graphs and equations represent relationships between variables.

## Essential Questions:

- How does the structure of equations and/or inequalities help us solve equations and/or inequalities?
- How does the substitution process help in solving problems?
- Why are variables used in equations? – What might a variable represent in a given situation?
- How are inequalities represented and solved?

## Mathematical Practices: (Practices to be explicitly emphasized are indicated with an *.)

1. **Make sense of problems and persevere in solving them.** Students choose the appropriate algebraic representations for given contexts and can create contexts given equations or inequalities.

2. **Reason abstractly and quantitatively.** Students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.

3. **Construct viable arguments and critique the reasoning of others.** Students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, graphs, and tables.

4. **Model with mathematics.** Students model problem situations in symbolic, graphic, tabular, and contextual formats. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and visual representations.

5. **Use appropriate tools strategically.**

6. **Attend to precision.** Students precisely define variables.

7. **Look for and make use of structure.** Students seek patterns or structures to model and solve problems using tables, equations and inequalities. Students apply properties to generate equivalent expressions (i.e. \(6 + 2x = 3 (2 + x)\) by distributive property) and solve equations (i.e. \(2c + 3 = 15, 2c = 12\) by subtraction property of equality, \(c = 6\) by division property of equality).

8. **Look for and express regularity in repeated reasoning.** Students generalize effective processes for representing and solving equations and inequalities based upon experiences.
## Grade 6: Unit 5: Equations and Inequalities

### Prerequisite Skills/Concepts:

*Students should already be able to:*

- Use variables in expressions and equations.
- Add, subtract, multiply and divide whole numbers, decimals and fractions.

### Advanced Skills/Concepts:

*Some students may be ready to:*

- Use properties of operations to create equivalent numerical expressions.
- Solve multi-step problems using rational numbers with expressions, equations and inequalities.
- Compare word problems and develop solution strategies by comparing the variable and number relationships in the situations.
- Recognize that multiplying or dividing an inequality by a negative number reverses the order of the comparison, hence the changes in what is positive or negative.

### Knowledge:  *Students will know...*

All standards in this unit go beyond the knowledge level.

### Skills:  *Students will be able to ...*

- Recognize that solving an equation or inequality is a process of answering a question: which values from a specified set, if any, make the equation or inequality true? (6.EE.5)
- Determine whether a given number in a specified set makes an equation or inequality true with substitution. (6.EE.5)
- Write variable expressions when solving a mathematical problem or real-world problem, recognizing that a variable can represent an unknown number or any number in a specified set (6.EE.6)
- Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which $p$, $q$ and $x$ are all nonnegative rational numbers. (6.EE.7)
- Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a mathematical problem or a real-world problem. (6.EE.8)
- Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions. (6.EE.8)
- Represent solutions of inequalities on number line diagrams. (6.EE.8)
<table>
<thead>
<tr>
<th>Grade 6: Unit 5: Equations and Inequalities</th>
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</thead>
<tbody>
<tr>
<td><strong>Academic Vocabulary:</strong></td>
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<tr>
<td><strong>Critical Terms:</strong></td>
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<tr>
<td>Infinite</td>
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<tr>
<td>Inequalities</td>
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<tr>
<td>Equations</td>
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<tr>
<td>Variables</td>
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<tr>
<td>Analyze</td>
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<tr>
<td>Substitution</td>
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<tr>
<td><strong>Supplemental Terms:</strong></td>
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<tr>
<td>Expression</td>
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<tr>
<td>Number line diagram</td>
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<td>Greater than &gt;</td>
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<td>Less than &lt;</td>
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<td>Greater than or equal to ≥</td>
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Connections to Previous Learning:
In grade 3, students were introduced to area of rectangles. In grade 5, students were introduced to volume of right rectangular prisms with whole number side lengths. They also learned how to multiply fractions and mixed numbers. In this unit, students will combine both of these concepts to determine the volume of right rectangular prisms with fractional edge lengths. When studying volume in grade 5, students packed prisms with unit cubes. They will be expected to continue using this strategy to justify that volume is the same as it would be when multiplying the lengths of the sides.

Focus of this Unit:
In grade 6, students are expected to apply their understanding of area to triangles, special quadrilaterals and composite figures. In addition, they will apply this skill to find surface area of 3-dimensional figures composed of triangles and rectangles using nets.

Connections to Subsequent Learning:
Students extend their understandings of area and surface area developed in Grade 6 to find surface areas of other polyhedra in Grade 7. They also extend their understandings of volume of rectangular prisms to solve problems involving volume of other three-dimensional figures.

From K-6, Geometry Progression document p.18-19:
Using the shape composition and decomposition skills acquired in earlier grades, students learn to develop area formulas for parallelograms, then triangles. They learn how to address three different cases for triangles: a height that is a side of a right angle, a height that “lies over the base” and a height that is outside the triangle.

Through such activity, students learn that any side of a triangle can be considered as a base and the choice of base determines the height (thus, the base is not necessarily horizontal and the height is not always in the interior of the triangle). The ability to view a triangle as part of a parallelogram composed of two copies of that triangle and the understanding that area is additive (see the Geometric Measurement Progression) provides a justification (MP3) for halving the product of the base times the height, helping students guard against the common error of forgetting to take half.

Also building on their knowledge of composition and decomposition, students decompose rectilinear polygons into rectangles, and decompose special quadrilaterals and other polygons into triangles and other shapes, using such decompositions to determine their areas, and justifying and finding relationships among the formulas for the areas of different polygons.

Building on the knowledge of volume (see the Geometric Measurement Progression) and spatial structuring abilities developed in earlier grades, students learn to find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism.
Students also analyze and compose and decompose polyhedral solids. They describe the shapes of the faces, as well as the number of faces, edges, and vertices. They make and use drawings of solid shapes and learn that solid shapes have an outer surface as well as an interior. They develop visualization skills connected to their mathematical concepts as they recognize the existence of, and visualize components of three-dimensional shapes that are not visible from a given viewpoint (MP1). They measure the attributes of these shapes, allowing them to apply area formulas to solve surface area problems (MP7). They solve problems that require them to distinguish between unit used to measure volume and units used to measure area (or length). They learn to plan the construction of complex three-dimensional compositions through the creation of corresponding two-dimensional nets (e.g., through a process of digital fabrication and/or graph paper. For example, they may design a living quarters (e.g., space station) consistent with given specifications for surface area and volume (MP2, MP7). In this and many other contexts, students learn to apply these strategies and formulas for areas and volumes to the solution of real-world and mathematical problems. These problems include those in which areas or volumes are to be found from lengths or lengths are to be found from volumes or areas and lengths. 

Students extend their understanding of properties of two-dimensional shapes to use of coordinate systems. For example, they may specify coordinates for a polygon with specific properties, justifying the attribution of those properties through reference to relationships among the coordinates (e.g., justifying that a shape is a parallelogram by comparing the lengths of its pairs of horizontal and vertical sides). 

As a precursor for learning to describe cross sections of three dimensional figures, students use drawings and physical models to learn to identify parallel lines in three-dimensional shapes, as well as lines perpendicular to a plane, lines parallel to a plane, the plane passing through three given points, and the plane perpendicular to a given line at a given point.

From K-6, Geometry Progression document p.20:
Composition and decomposition of shapes is used throughout geometry from Grade 6 to high school and beyond. Composition and decomposition of regions continues to be important for solving a wide variety of area problems, including justifications of formulas and solving real world problems that involve complex shapes.
## Desired Outcomes

### Standard(s):

Solve real-world and mathematical problems involving area, surface area, and volume.

- **6.G.1** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
- **6.G.2** Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas \( V = lwh \) and \( V = bh \) to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
- **6.G.4** Represent three-dimensional figures using nets made up of rectangles and triangles. Use the nets to find surface areas of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

### WIDA Standard: (English Language Learners)

English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics. English language learners will benefit from:

- the use of manipulatives and visuals when decomposing 3-dimensional figures into nets.
- using unit cubes to study volume of prisms.
- explicit vocabulary instruction for the types, components and measurement units of geometric figures.

### Understandings: Students will understand that ...

- Geometry and spatial sense offer ways to envision, to interpret and to reflect on the world around us.
- Area, volume and surface area are measurements that relate to each other and apply to objects and events in our real life experiences.
- Properties of 2-dimensional shapes are used in solving problems involving 3-dimensional shapes.
- The value of numbers and application of properties are used to solve problems about our world.

### Essential Questions:

- How does what we measure influence how we measure?
- How can space be defined through numbers and measurement?
- How does investigating figures help us build our understanding of mathematics?
- What is the relationship between 2-dimensional shapes, 3-dimensional shapes and our world?
Mathematical Practices: (Practices to be explicitly emphasized are indicated with an *.)

1. **Make sense of problems and persevere in solving them.** Given a three dimensional figure a student will solve for the surface area using the formula or the net with fractional edges and be able to use resources, independently.

2. **Reason abstractly and quantitatively** – Students will use their understanding of the value of fractions in solving with area. Students will be able to see and justify the reasoning for decomposing and composing of an irregular polygon/nets using area of triangles and quadrilaterals to solve for surface area. Students will use the relationships between two-dimensional and three-dimensional shapes to understand surface area.

3. **Construct viable arguments and critique the reasoning of others.** Students will be able to review solutions to justify (verbally and written) why the solutions are reasonable.

4. **Model with mathematics.** Use hands on/virtual manipulatives (prisms, pyramids and folding nets) using every day two-dimensional and three-dimensional shapes.

5. **Use appropriate tools strategically.** Students will use a ruler, graph paper two-dimensional and three-dimensional shapes to solve for area, volume and surface area. In addition, students will determine appropriate formulas to use for given situations.

6. **Attend to precision.** Students will use appropriate measurement units (square units vs. cubic units) and correct terminology to justify reasonable solutions.

7. **Look for and make use of structure.** Students will understand the relationship between the structure of a three-dimensional shape and its volume formula. Students also decompose two-dimensional figures to find areas.

8. **Look for and express regularity in repeated reasoning.** Students will explain why formula or process is used to solve given problems. Students use properties of figures and properties of operations to connect formulas to surface area and volume.

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**Prerequisite Skills/Concepts:**

*Students should already be able to:*

- Geometric Measurement: understand concepts of volume and relate volume to multiplication and to addition.
- Perform operations with multi-digit whole numbers and with decimals to hundredths.
- Solve problems involving multiplication of fractions and mixed numbers.

---

**Advanced Skills/Concepts:**

*Some students may be ready to:*

- Derive formulas for volume of pyramids and non-rectangular prisms.
<table>
<thead>
<tr>
<th>Knowledge: <strong>Students will know...</strong></th>
<th>Skills: <strong>Students will be able to ...</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>• Formula for volume of a right rectangular prism.</td>
<td>• Given irregular figures, students will be able to divide the shape into triangles and rectangles (6.G.1)</td>
</tr>
<tr>
<td>• Procedures for finding surface area of pyramids and prisms.</td>
<td>• Given a polygon, students will find the area using the decomposing shapes. (6.G.1)</td>
</tr>
<tr>
<td></td>
<td>• Given a polygon students will calculate the area by decomposing into composite figures (triangles and rectangles). (6.G.1)</td>
</tr>
<tr>
<td></td>
<td>• Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. (6.G.2)</td>
</tr>
<tr>
<td></td>
<td>• Calculate the volume of a right rectangular prism. (6.G.2)</td>
</tr>
<tr>
<td></td>
<td>• Apply the formula to solve real world mathematical problems involving volume with fractional edge lengths. (6.G.2)</td>
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<tr>
<td></td>
<td>• Represent 3D figures using nets of triangles and rectangles. (6.G.4)</td>
</tr>
<tr>
<td></td>
<td>• Solve real world problems involving surface areas using nets. (6.G.4)</td>
</tr>
</tbody>
</table>
## Grade 6: Unit 6: Geometry

### Academic Vocabulary:

<table>
<thead>
<tr>
<th>Critical Terms:</th>
<th>Supplemental Terms:</th>
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<tbody>
<tr>
<td>Net</td>
<td>Polygon</td>
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<tr>
<td>Surface Area</td>
<td>Quadrilateral</td>
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<td></td>
<td>Rectangle</td>
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<td></td>
<td>Triangle</td>
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<td>Trapezoid</td>
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<td>Area</td>
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<td></td>
<td>Volume</td>
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<td>Rectangular Prism</td>
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<td></td>
<td>Decomposing</td>
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<td></td>
<td>Vertex</td>
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<td></td>
<td>Face</td>
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<td></td>
<td>Edge</td>
</tr>
<tr>
<td></td>
<td>Rhombus</td>
</tr>
<tr>
<td></td>
<td>Right angle</td>
</tr>
<tr>
<td></td>
<td>Kites</td>
</tr>
</tbody>
</table>

**Priority Standards** = Approximately 70%

**Supporting Standards** = Approximately 20%

**Additional Standards** = Approximately 10%
Connections to previous learning:
Students have experiences with data gathered from measurement.

Focus Within Grade Level:
Students develop a sense of statistical variability, summarizing and describing distributions. Students gain experience doing investigations, especially statistical investigations, by starting with a question. The data gathered to answer the question is interpreted in light of the variability of the data relative to the situation where the data resides, the question being asked and how the data is distributed over the data set. Whether larger numbers such as those involving populations of states or small, such as the changes in plant height over a week, the variability of the data matters. Students learn to make histogram and box plot data displays, and further their expertise with dot plots (line plots) when working with measurements or quantities that are counted. The shape of displayed data, especially symmetry, is considered in analysis of data distributions, including the identification of clusters, peaks and gaps. Measures of central tendency and spread, including median, quartiles, the interquartile range, are used.

Connections to Subsequent Learning:
Following the idea of statistical variability, in seventh grade, students are introduced to ideas of randomness, probability, random sampling and comparison of populations.

From the 6-8 Statistics and Probability Progression Document, pp. 4-6:
**Develop understanding of statistical variability:** Statistical investigations begin with a question, and students now see that answers to such questions always involve variability in the data collected to answer them. Variability may seem large, as in the selling prices of houses, or small, as in repeated measurements on the diameter of a tennis ball, but it is important to interpret variability in terms of the situation under study, the question being asked, and other aspects of the data distribution. A collection of test scores that vary only about three percentage points from 90% as compared to scores that vary ten points from 70% lead to quite different interpretations by the teacher. Test scores varying by only three points is often a good situation. But what about the same phenomenon in a different context: percentage of active ingredient in a prescription drug varying by three percentage points from order to order?

Working with counts or measurements, students display data with the dot plots (sometimes called line plots) that they used in earlier grades. New at Grade 6 is the use of histograms, which are especially appropriate for large data sets.

Students extend their knowledge of symmetric shapes, to describe data displayed in dot plots and histograms in terms of symmetry. They identify clusters, peaks, and gaps, recognizing common shapes and patterns in these displays of data distributions.
A major focus of Grade 6 is characterization of data distributions by measures of center and spread. To be useful, center and spread must have well-defined numerical descriptions that are commonly understood by those using the results of a statistical investigation. The simpler ones to calculate and interpret are those based on counting. In that spirit, center is measured by the median; a number arrived at by counting to the middle of an ordered array of numerical data. When the number of data points is odd, the median is the middle value. When the number of data points is even, the median is the average of the two middle values. Quartiles, the medians of the lower and upper halves of the ordered data values, mark off the middle 50% of the data values and, thus, provide information on the spread of the data. The distance between the first and third quartiles, the interquartile range (IQR), is a single number summary that serves as a very useful measure of variability.

Plotting the extreme values, the quartiles, and the median (the five-number summary) on a number line diagram, leads to the box plot, a concise way of representing the main features of a data distribution. Box plots are particularly well suited for comparing two or more data sets, such as the lengths of mung bean sprouts for plants with no direct sunlight versus the lengths for plants with four hours of direct sunlight per day.

Students use their knowledge of division, fractions, and decimals in computing a new measure of center—the arithmetic mean, often simply called the mean. They see the mean as a “leveling out” of the data in the sense of a unit rate (see Ratio and Proportion Progression). In this “leveling out” interpretation, the mean is often called the “average” and can be considered in terms of “fair share.” For example, if it costs a total of $40 for five students to go to lunch together and they decide to pay equal shares of the cost, then each student’s share is $8.00. Students recognize the mean as a convenient summary statistic that is used extensively in the world around them, such as average score on an exam, mean temperature for the day, average height and weight of a person of their age, and so on.

Students also learn some of the subtleties of working with the mean, such as its sensitivity to changes in data values and its tendency to be pulled toward an extreme value, much more so than the median. Students gain experience in deciding whether the mean or the median is the better measure of center in the context of the question posed. Which measure will tend to be closer to where the data on prices of a new pair of jeans actually cluster? Why does your teacher report the mean score on the last exam? Why does your science teacher say, “Take three measurements and report the average?”
For distributions in which the mean is the better measure of center, variation is commonly measured in terms of how far the data values deviate from the mean. Students calculate how far each value is above or below the mean, and these deviations from the mean are the first step in building a measure of variation based on spread to either side of center. The average of the deviations is always zero, but averaging the absolute values of the deviations leads to a measure of variation that is useful in characterizing the spread of a data distribution and in comparing distributions. This measure is called the mean absolute deviation, or MAD. Exploring variation with the MAD sets the stage for introducing the standard deviation in high school.

Summarize and describe distributions: “How many text messages do middle school students send in a typical day?”

Data obtained from a sample of students may have a distribution with a few very large values, showing a “long tail” in the direction of the larger values. Students realize that the mean may not represent the largest cluster of data points, and that the median is a more useful measure of center. In like fashion, the IQR is a more useful measure of spread, giving the spread of the middle 50% of the data points.

The 37 animal speeds shown can be used to illustrate summarizing a distribution. According to the source, “Most of the following measurements are for maximum speeds over approximate quarter-mile distances. Exceptions—which are included to give a wide range of animals—are the lion and elephant, whose speeds were clocked in the act of charging; the whippet, which was timed over a 200-yard course; the cheetah over a 100-yard distance; humans for a 15-yard segment of a 100-yard run; and the black mamba snake, six-lined race runner, spider, giant tortoise, three toed sloth, . . . , which were measured over various small distances.” Understanding that it is difficult to measure speeds of wild animals, does this description raise any questions about whether or not this is a fair comparison of the speeds?

Moving ahead with the analysis, students will notice that the distribution is not symmetric, but the lack of symmetry is mild. It is most appropriate to measure center with the median of 35 mph and spread with the IQR of 42 - 25 = 17. That makes the cheetah an outlier with respect to speed, but notice again the description of how this speed was measured. If the garden snail with a speed of 0.03 mph is added to the data set, then cheetah is no longer considered an outlier. Why is that?

Because the lack of symmetry is not severe, the mean (32.15 mph) is close to the median and the MAD (12.56 mph) is a reasonable measure of typical variation from the mean, as about 57% of the data values lie within one MAD of the mean, an interval from about 19.6 mph to 44.7 mph.
Grade 6: Unit 7: Statistics

Desired Outcomes

Standard(s):

Develop understanding of statistical variability

- **6.SP.1** Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example,* “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.

- **6.SP.2** Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.

- **6.SP.3** Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Summarize and describe distributions

- **6.SP.4** Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

- **6.SP.5** Summarize numerical data sets in relation to their context, such as by:
  
  a) Reporting the number of observations.
  
  b) Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
  
  c) Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
  
  d) Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Supporting Standards

Understand ratio concepts and use ratio reasoning to solve problems

- **6.RP.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
  
  b) Solve unit rate problems including those involving unit pricing and constant speed. *For example,* if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? *At what rate were lawns being mowed?*

WIDA Standard: (English Language Learners)

English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics. English language learners benefit from:

- explicit instruction with regard to the components of visual and symbolic data representations.
- explicit vocabulary instruction and attention to units represented in data distributions.
- hands-on activities to experience data collection (such as coin flips, spinners, dice rolls, etc.).

Priority Standards = Approximately 70%
Supporting Standards = Approximately 20%
Additional Standards = Approximately 10%
### Understandings: Students will understand that...
- Statistical questions and the answers account for variability in the data.
- The distribution of a data set is described by its center, spread, and overall shape.
- Measures of center for a numerical set of data are summaries of the values using a single number.
- Measures of variability describe the variation of the values in the data set using a single number.

### Essential Questions:
- How do we analyze and interpret data sets?
- When is one data display better than another? How do mathematicians choose to display data in strategic ways?
- When is one statistical measure better than another?
- What makes a good statistical question?

### Mathematical Practices: (Practices to be explicitly emphasized are indicated with an *.)

1. **Make sense of problems and persevere in solving them.** Students will make sense of the data distributions by interpreting the measures of center and variability in the context of the situations they represent.
2. **Reason abstractly and quantitatively.** Students reason about the appropriate measures of center or variability to represent a data distribution.
3. **Construct viable arguments and critique the reasoning of others.** Students construct arguments regarding which measures of center or variability they would use to represent a particular data distribution. They may critique other students' choices when considering how outliers are handled in each situation.
4. **Model with mathematics.** Students begin to explore covariance and represent two quantities simultaneously. They use measures of center and variability and data displays (i.e. box plots and histograms) to draw inferences about and make comparisons between data sets. Students need many opportunities to connect and explain the connections between the different representations. Students collect data regarding real-world contexts and create models to display and interpret the data.
5. **Use appropriate tools strategically.** Students consider available tools (including estimation and technology) when answering questions about data or representing data distributions. They decide when certain tools might be helpful. For instance, students in grade 6 may decide to represent similar data sets using dot plots with the same scale to visually compare the center and variability of the data.
6. **Attend to precision.** Students use appropriate terminology when referring data displays and statistical measures.
7. **Look for and make use of structure.** Students examine the structure of data representations by examining intervals, units, and scale in box plots, line plots, histograms and dot plots.
8. **Look for and express regularity in repeated reasoning.** Students recognize typical situations in which outliers skew data. They can explain patterns in the way data is interpreted in the various representations they study throughout this unit.
### Grade 6: Unit 7: Statistics

#### Prerequisite Skills/Concepts:

*Students should already be able to:*

View statistical reasoning as a four-step investigative process:
1. Formulate questions that can be answered with data.
2. Design and use a plan to collect relevant data.
3. Analyze the data with appropriate methods.
4. Interpret results and draw valid conclusions from the data that relate to the questions posed.

#### Advanced Skills/Concepts:

*Some students may be ready to:*

- Examine and compare measures of center and variability for random samples.

#### Knowledge:  *Students will know...*

- Median and mean are measures of center.
- Interquartile range and mean absolute deviation are measures of variability.
- The distribution is the arrangement of the values in a data set.

#### Skills:  *Students will be able to ...*

- Identify statistical questions. (6.SP.1)
- Determine if questions anticipate variability in the data related to the question and account for it in the answers. (6.SP.1)
- Represent a set of data collected to answer a statistical question and describe it by its center, spread, and overall shape. (6.SP.2)
- Represent and explain the difference between measures of center and measures of variability. (6.SP.3)
- Display numerical data in plots on a number line. (6.SP.4)
- Display numerical data in dot plots. (6.SP.4).
- Display numerical data in histograms. (6.SP.4)
- Display numerical data in box plots. (6.SP.4)
- Use language to summarize numerical data sets in relation to their context. (6.SP.5)
- Report the number of observations. (6.SP.5)
- Describe the nature of the attribute under investigation. (6.SP.5)
- Give quantitative measures of center and variability as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. (6.SP.5)
- Relate the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered. (6.SP.5)
### Academic Vocabulary:

<table>
<thead>
<tr>
<th>Critical Terms:</th>
<th>Supplemental Terms:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measure of Variation</td>
<td>Mean absolute deviation</td>
</tr>
<tr>
<td>Number line</td>
<td>Cluster</td>
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<tr>
<td>Dot plot</td>
<td>Peak</td>
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<tr>
<td>Histogram</td>
<td>Gap</td>
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<tr>
<td>Box plot</td>
<td>Frequency table</td>
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<tr>
<td>Data Sets</td>
<td>Symmetrical quartile</td>
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<tr>
<td>Median</td>
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<tr>
<td>Mean</td>
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<td>Striking deviation</td>
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<tr>
<td>Outliers</td>
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<td>Measures of center</td>
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<tr>
<td>Variability</td>
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<tr>
<td>Data</td>
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<tr>
<td>Interquartile range</td>
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<tr>
<td>Distribution</td>
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<tr>
<td>Skew</td>
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Connections to Previous Learning:
Previously, students learned to find the area of shapes in 3rd grade, progressing to the area of a net for rectangular solids and the volume of solids as indicated in the measurement standards for K-5. It is expected that students know how to add, subtract and multiply with fractional amounts. Learning about expressions and equations in the previous units of 6th grade provides background on solving equations, the meaning of variable and making connections between expressions and equations, graphs and a real world or mathematical situations. Students already have learned the foundational ideas of data distributions, measures of center and variability, which can be displayed in visual formats.

Focus of this Unit:
In this unit students will solve a variety of real world problems and communicate solutions using dot plots. They will apply their understanding of graphing on the coordinate plane to a study of the relationship between side lengths and volume. Students will determine which variables are dependent and independent in this real-world context and will create expressions/equations based on their analysis of their data and variables. Problems involving geometric shapes, like finding the volume of a right rectangular prism, will also be solved by students using their knowledge of variables and graphing techniques. For instance, students practice using numbers to describe relationships between quantities that vary together to identify and understand the relationship between the side lengths of a box and the box’s volume where the net for the box is described on a 4 quadrant grid. Students use coordinate geometry in a four-quadrant grid to determine lengths, and develop deeper understanding of the volume formula for a rectangular solid.

Connections to Subsequent Learning:
The focus on the relationship between two quantities and the equation leads to work with functions and provides examples of applications for the idea of variable. In 7th grade, students continue to construct and describe the relationships between geometrical figures and solve problems with volume.
## Desired Outcomes

**Standard(s):**

**Represent and analyze quantitative relationships between dependent and independent variables.**

- **6.EE.9** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation \( d = 65t \) to represent the relationship between distance and time.*

**Apply and extend previous understandings of numbers to the system of rational numbers.**

- **6.NS.8** Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

**Summarize and describe distributions.**

- **6.SP.4** Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

**Solve real-world and mathematical problems involving area, surface area, and volume.**

- **6.G.2** Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas \( V = l \times w \times h \) and \( V = b \times h \) to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

**Supporting Standards:**

**Solve real-world and mathematical problems involving area, surface area, and volume.**

- **6.G.3** Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

- **6.G.4** Represent three-dimensional figures using nets made up of rectangles and triangles. Use the nets to find surface areas of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

**WIDA Standard: (English Language Learners)**

*English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics. English language learners benefit from:*

- explicit instruction with regard to the context of the tasks for geometric measurement, data distributions, and dependent and independent variables.
- tactile and virtual tools for measuring dimensions of geometric figures and displaying data.
**Understandings:** *Students will understand that* …

- Graphing objects in a four quadrant graph can provide ways to measure distances and identify that shapes have specific properties.
- Expressions are a modeling tool to use when solving real word problems. This process can provide a way to describe quantitative relationships – for instance, traveling some distance (d) at a given rate of travel will take a given amount of time (t) with a constant rate.
- Volume of a rectangular prism can be determined by multiplying the length, width and height dimensions when the dimensions are fractional lengths.

**Essential Questions:**

- What is the relationship between the dimensions of a figure and its volume?
- How do ordered pairs on coordinate graphs help define relationships?
- How do we determine if a variable is independent or dependent in an expression or equation?
- What is the value of using different data representations?
- What models are helpful for understanding and quantifying the volume of rectangular prisms?

**Mathematical Practices:** (Practices to be explicitly emphasized are indicated with an *.)

1. **Make sense of problems and persevere in solving them.** Students apply understandings developed from previous experiences with expressions and equations to represent and solve problems regarding volume and data.
2. **Reason abstractly and quantitatively.** Students demonstrate abstract reasoning by using formulas to create expressions and equations with numbers and variables so that they can efficiently represent and solve problems.
3. **Construct viable arguments and critique the reasoning of others.** Students construct and critique arguments about effective algebraic, graphic and tabular representations about geometric figures and data distributions to solve problems.
4. **Model with mathematics.** Students model problem situations symbolically, graphically, using a table, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations.
5. **Use appropriate tools strategically.** Students choose formulas appropriate to given situations. They also choose appropriate algebraic and visual models to use when representing and solving problems.
6. **Attend to precision.** Students label numeric values in problem-solving situations and attend to the language of contextualize problems when creating models.
7. **Look for and make use of structure.** Students examine the geometric structures to determine appropriate problem solving strategies. They consider the structures of data representations when creating algebraic representations to solve problems. They use the structure of problems to differentiate between dependent and independent variables.
8. **Look for and express regularity in repeated reasoning.** Students generalize strategies based upon experiences with multiple problem solving structures, geometric problems and data distributions.
### Prerequisite Skills/Concepts:

**Students should already be able to:**
- Use variables to represent unknowns.
- Add, subtract, multiply and divide with whole numbers, decimals and fractions.

### Advanced Skills/Concepts:

**Some students may be ready to:**
- Find relationships between two quantities and the equation as related to work with functions.
- Construct and describe the relationships between geometrical figures and solve problems with volume fluently.

### Knowledge:  **Students will know...**

All standards in this unit go beyond the knowledge level.

### Skills:  **Students will be able to ...**

- Define independent and dependent variables. (6.EE.9)
- Use variables to represent two quantities in a real-world problem that change in relationship to one another. (6.EE.9)
- Write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. (6.EE.9)
- Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. (6.EE.9)
- Solve real-world problems by graphing points in all four quadrants of the coordinate plane. (6.NS.8)
- Use coordinates to find distances between points with the same first coordinate or the same second coordinate. (6.NS.8)
- Use absolute value to find distances between points with the same first coordinate or the same second coordinate. (6.NS.8)
- Display numerical data in plots on a number line, dot plots, histograms and box plots. (6.SP.4)
- Calculate the volume of a right rectangular prism. (6.G.2)
- Apply the formula to solve real world mathematical problems involving volume with fractional edge lengths. (6.G.2)
# Academic Vocabulary:

<table>
<thead>
<tr>
<th>Critical Terms:</th>
<th>Supplemental Terms:</th>
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<tbody>
<tr>
<td>Volume</td>
<td>Net</td>
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<tr>
<td>Independent variables</td>
<td>Edge</td>
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<tr>
<td>Dependent variables</td>
<td>Dot plot</td>
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<tr>
<td>Ordered pairs</td>
<td>Histogram</td>
</tr>
<tr>
<td>x-axis</td>
<td>Box plot</td>
</tr>
<tr>
<td>x coordinate</td>
<td>Rectangular prism</td>
</tr>
<tr>
<td>y-axis</td>
<td></td>
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<tr>
<td>y coordinate</td>
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